All order α' higher derivative corrections to non-BPS branes of type IIB Super string theory

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Abstract

By dealing with the evaluation of string theory correlators of $< V_C V_T V_{\bar{\Psi}} V_\Psi >$, the complete and closed form of the amplitude of two fermion fields, one tachyon and one closed string Ramond-Ramond field in type IIB superstring theory is found. Specifically by comparing infinite tachyon poles in field theory amplitude with infinite tachyon poles of the S-matrix of string amplitude (for $p + 1 = n$ case), all the infinite higher derivative corrections of two tachyons and two fermions (in type IIB) to all orders of α' have been discovered. Using these new couplings, we are able to produce infinite $t' + s' + u$-channel tachyon poles of string theory in field theory. Due to internal degrees of freedom of fermions and tachyon (Chan-Paton factors) we comment that, neither there should be single $s, t-$channel fermion (tachyon pole) nor their infinite poles.

Due to internal CP factor we also discover that there is no coupling between two closed string Ramond-Ramond (RR) field and one tachyon in type II super string theory.

Taking into account the fact that the kinetic terms of fermions, gauge, scalar fields and tachyons do not obtain any higher derivative corrections, and due to their CP factors, string theory amplitude dictates us there should not be any double poles in the amplitude of one RR, two fermions and one tachyon.

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1   Introduction

D_p-branes must be interpreted just as sources for Ramond-Ramond (closed string C-field) for both BPS and non-BPS branes [1, 2]. Applying Ramond-Ramond (RR) couplings various issues such as brane within branes [3, 4] , K-theory in the language of D-branes [5, 6], Myers effect [7, 8] and its all order corrections [9] have been well understood.

It is also worth trying to follow Born-Infeld action and its generalization which was appeared in [10, 11].

By dealing with unstable branes, one might hope to reveal some of the properties of type IIB (IIA) String theories within some special backgrounds (more precisely only time-dependent ones). For more explanations we refer to some of the basic references [12, 13, 14, 15, 16, 17, 18, 19, 20, 21].

Having used tachyonic action [22] and in particular, using its correct and complete form (based on S-Matrix calculations, including the internal degrees of freedom of tachyons in different pictures)[23], one is able to describe most of the known properties of the decay of the unstable D_p-branes (where spatial dimension of brane p is even (odd) for IIB (IIA) theories) just for stable point inside tachyon' s potential.

As argued in detail in [23] the higher derivative corrections of tachyonic action (including both non-BPS and D-brane anti D-brane effective actions) are indeed important around their unstable point. In section 2 of [23] we have made several aims and motivations for following unstable branes nevertheless let us just point out some of them once more.

As an instance for some holographic models with symmetry breaking in QCD one has to use tachyonic action and its higher derivative corrections [24, 25]. The other motivation is as follows.

In order to deal with inflation in string theory [26, 27, 28] , one has to make use of the effective action of brane -anti brane system involving its correct higher derivative corrections [29, 30]. For more explanations section 2 of [23] is realized. For the other applications on non-BPS branes one might look at [31].

One way for finding these higher derivative corrections is to apply directly the scattering theory of non-BPS branes. Indeed by applying this method we are able to actually explore new couplings including their corrections and essentially in a very exact manner their

coefficients can be fixed.

However due to S-matrix formalism, around the unstable point of tachyonic DBI action, the internal degrees of freedom of tachyons must be taken into account, namely open string tachyons in (-1)-picture do carry σ_2 and in zero picture they carry σ_1 Pauli matrix.

On the other hand we have constructed and checked the exact conditions for the other part of the effective actions, basically the Wess-Zumino part of both brane anti brane and non-BPS branes will give rise all order terms of the resulting amplitudes.

For good reasons we would like to highlight the main works [32, 33, 29, 34]. Notice that just some of these couplings can be discovered by making use of BSFT formalism [35].

In order to observe some of the applications for all the infinite higher derivative corrections on BPS branes, N^3 entropy of $M5$ branes (regarding Dielectric effect and Black brane entropy growth) [36] has been suggested. Concerning the applications on M-theory [37, 38] may be considered.

It is worth trying to talk about super symmetrized version of tachyonic DBI action. The fermions are also embedded in tachyonic DBI action [39]. Having removed tachyons in this action , one may re-derive the super symmetric version of DBI action (for further details [40, 41] should be considered).

From now on we would like to keep tachyons for our explicit computations. Keeping them fixed to the action (the static gauge and special normalization for their kinetic term are employed), we obtain the Lagrangian as follows

$$L = -T_p V(T) \sqrt{-\det(\eta_{ab} + 2\pi\alpha' F_{ab} - 2\pi\alpha'\bar{\Psi}\gamma_b\partial_a\Psi + \pi^2\alpha'^2\bar{\Psi}\gamma^\mu\partial_a\Psi\bar{\Psi}\gamma_\mu\partial_b\Psi + 2\pi\alpha'\partial_a T\partial_b T)}$$

Remarks on tachyon potential with different approaches have been mentioned (see [42, 43, 23]), however it is good to know that the potential which is used to work in the S-matrix computations in super string theory is

$$V(T) = e^{-\pi TT/2}$$

In particular its expansion up to fourth order makes consistent results with scattering amplitude arguments. The other point should be made, is that as tachyon goes to infinity the term inside the effective action $T^4 V(TT)$ tends to zero as we have already expected from a single unstable brane's condensation.

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All the infinite non-Abelian higher derivative corrections of two fermions two tachyons are not embedded into this action. Here we would like to perform the S-matrix calculations at disk level of one closed string Ramond-Ramond (C-field), one tachyon and two world volume fermions in type IIB super string theory to explore non-Abelian couplings of two fermions and two tachyons to all orders in α'. Obviously by looking for two fermion two tachyon 's amplitude one can fix the needed coefficients in field theory as we will go through them in detail. An important point has to be clarified as follows:

By carrying out just this S-matrix ($CT\bar{\psi}\psi$), one can precisely fix the coefficients of all higher derivative corrections of two fermions two tachyons in the world volume of non-BPS branes.

Note that there should not be any coupling between two fermions and one tachyon in the world volume of non-BPS branes as we comment it later on.

Therefore we come to the important fact for which this four point (technically five point) function ($CT\bar{\psi}\psi$), is the only exception so far which does not include any s, t, u -channel poles.

2   Notations and remarks on scattering of non-BPS branes

Before moving to finding out our amplitude, mentioning some remarks on the scattering of non-BPS branes is inevitable. Apart from some needed notations, here we want to show that although the amplitude of two world volume fermions and one tachyon in II string theories has non-zero value, however such a coupling in field theory is not certainly allowed.

The reason for this conclusion is that both kinetic terms of fermion fields and tachyons have to carry two fermions and two tachyons. We come over to this problem by relabeling the internal degrees of freedom to both left and right hand side of the vertex operator of Ramond field.

It is worth to address some of the works which have been completely dealt with scattering amplitudes of non-BPS [32, 34, 23, 30] and BPS branes [44, 45, 46, 47, 48, 49, 50, 9, 51, 52] at tree level computations.

To achieve our goals, we need to remind the structure of the needed vertex operators [2]

$$
\begin{aligned}
V_T^{(0)}(x) &= \alpha' ik \cdot \psi(x) e^{\alpha' ik \cdot X(x)} \\
V_T^{(-1)}(x) &= e^{-\phi(x)} e^{\alpha' ik \cdot X(x)} \\
V_{\bar{\Psi}}^{(-1/2)}(x) &= \bar{u}^A e^{-\phi(x)/2} S_A(x) \, e^{\alpha' iq \cdot X(x)} \\
V_{\Psi}^{(-1/2)}(x) &= u^B e^{-\phi(x)/2} S_B(x) \, e^{\alpha' iq \cdot X(x)} \\
V_C^{(-\frac{1}{2},-\frac{1}{2})}(z,\bar{z}) &= (P_- \slashed{H}_{(n)} M_p)^{\alpha\beta} e^{-\phi(z)/2} S_\alpha(z) e^{i\frac{\alpha'}{2} p \cdot X(z)} e^{-\phi(\bar{z})/2} S_\beta(\bar{z}) e^{i\frac{\alpha'}{2} p \cdot D \cdot X(\bar{z})} \quad (1)
\end{aligned}
$$

The on-shell conditions for all strings, including fermions, Ramond-Ramond and tachyon are $q^2 = p^2 = 0, k^2 = \frac{1}{2\alpha'}$. Majorana-Weyl $(\bar{u}^A, u^B)$ function in all ten dimensions have been introduced in [52]. It has already been pointed out that one has to employ charge conjugation $C^{\alpha\beta}$ as well. Let us just emphasize on the structure of RR's field strength with $n = 1, 3, 5$ and $a_n = 1$ in type IIB super string theory as follows

$$
\slashed{H}_{(n)} = \frac{a_n}{n!} H_{\mu_1 \dots \mu_n} \gamma^{\mu_1} \dots \gamma^{\mu_n} \ ,
$$

A useful trick (doubling trick) in order just to make use of the holomorphic correlations has been applied. Further notations can be obtained in [23, 52].

Notice to the important point that the vertices of two fermion fields are similar to the vertex of a closed string Ramond-Ramond field so one may expect that the amplitude of two fermions and one tachyon can be derived from the amplitude of one RR and one tachyon in the world volume of non-BPS branes, however due to their internal Chan-Paton factors we comment that it is no longer true.

The amplitude of two fermions and one tachyon in the world volume of type II string theory without taking into account the internal degrees of freedom (Chan-Paton factors) is given as follows:

$$
\mathcal{A}^{\bar{\psi}\psi T} \sim \int dx_1 dx_2 dx_3 \langle V_{\bar{\psi}}^{(-1/2)}(x_1) V_{\psi}^{(-1/2)}(x_2) V_T^{(-1)}(x_3) \rangle, \quad (2)
$$

Having replaced the mentioned vertex operators in the amplitude, we reach to

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[2]here we keep α' and for the moment do not talk about internal CP degrees of freedom of the vertex operators

$$\mathcal{A}^{\bar{\psi},\psi,T} \sim \int dx_1 dx_2 dx_3 x_{12}^{-1/4}(x_{13}x_{23})^{-1/2}|x_{12}|^{\alpha'^2 k_1.k_2}|x_{13}|^{\alpha'^2 k_1.k_3}|x_{23}|^{\alpha'^2 k_2.k_3}$$
$$\times <: S_A(x_1) : S_B(x_2) :> \bar{u}_1^A u_2^B \tag{3}$$

using

$$<: S_A(x_1) : S_B(x_2) :> = x_{12}^{-5/4}(C^{-1})_{AB}.$$

and applying the holomorphic correlators for all the fields

$$\langle X^\mu(z)X^\nu(w)\rangle = -\frac{\alpha'}{2}\eta^{\mu\nu}\log(z-w) ,$$
$$\langle \psi^\mu(z)\psi^\nu(w)\rangle = -\frac{\alpha'}{2}\eta^{\mu\nu}(z-w)^{-1} ,$$
$$\langle \phi(z)\phi(w)\rangle = -\log(z-w) . \tag{4}$$

one can explicitly check the $SL(2,R)$ invariance of this amplitude. Gauge fixing as $(x_1, x_2, x_3) = (0, 1, \infty)$ into (3), the amplitude should be read as

$$\mathcal{A}^{\bar{\Psi},\Psi,T} = \bar{u}_1^A C_{AB}^{-1} u_2^B \left(\text{Tr}\,(\lambda_1\lambda_2\lambda_3) - \text{Tr}\,(\lambda_1\lambda_3\lambda_2)\right) \tag{5}$$

which has non-zero value, however the final result of string theory can not be reproduced in field theory by extracting the kinetic term of the fermion fields $(2\pi\alpha'\bar{\psi}\gamma^\mu D_\mu\psi)$.

Also note that the tachyon potential is an even function of tachyon field and also in the DBI effective action both fermion and tachyon fields are even functions, so we come to the fact that this amplitude should have zero result in field theory as well as in string theory side.

The only possibility for not having such a coupling is to devote Chan-Paton factors to fermion fields.

The internal degree of freedom of massless gauge field in (-1)-picture for non-BPS branes has already been fixed (to be σ_3) in [23]. Since the amplitude of two fermions and one gauge field has non zero value even for non-BPS branes (as this has been calculated and shown that it has certainly non zero value [53]) , we come to the fact that the internal multiplication of CP factors of two fermion fields must carry σ_3 in order for getting non zero value for the amplitude of $A^{\bar{\psi}^{-1/2}\psi^{-1/2}A^{-1}}$.

Hence one can postulate this internal degree of freedom (σ_3) to right hand side of massless Ramond vertex operator and apply picture changing operator (which carries σ_3 CP factor [43]) on it to get CP factor of left hand side of the Ramond vertex operator which turned out to be an identity matrix (albeit there is an ambiguity about devoting the CP factor of σ_3 to right hand side or left hand side of Ramond vertex operator, however for our purpose we need not worry about this ambiguity).

On the other hand the CT amplitude in the world volume of non-BPS branes has certainly non zero value such that by fixing $(x_1 = \infty, z = i, \bar{z} = -i)$ we obtain the final form of the amplitude of one closed string RR and one tachyon as

$$A^{CT} = (2i)\text{Tr}\,(P_- \slashed{H}_{(n)} M_p)$$

This result should be reproduced by taking into account the following coupling

$$2i\beta'\mu'_p(2\pi\alpha') \int_{\Sigma_{p+1}} C_p \wedge DT$$

Therefore we need to postulate the following CP factors to all vertices in the presence of non-BPS branes

$$
\begin{aligned}
V_T^{(0)}(x) &= \alpha' ik\cdot\psi(x)e^{\alpha' ik\cdot X(x)}\lambda \otimes \sigma_1, \\
V_T^{(-1)}(x) &= e^{-\phi(x)}e^{\alpha' ik\cdot X(x)}\lambda \otimes \sigma_2 \\
V_{\bar{\Psi}}^{(-1/2)}(x) &= \bar{u}^A e^{-\phi(x)/2}S_A(x)\,e^{\alpha' iq\cdot X(x)}\lambda \otimes \sigma_3 \\
V_{\Psi}^{(-1/2)}(x) &= u^B e^{-\phi(x)/2}S_B(x)\,e^{\alpha' iq\cdot X(x)}\lambda \otimes I \\
V_C^{(-\frac{1}{2},-\frac{1}{2})}(z,\bar{z}) &= (P_-\slashed{H}_{(n)}M_p)^{\alpha\beta}e^{-\phi(z)/2}S_\alpha(z)e^{i\frac{\alpha'}{2}p\cdot X(z)}e^{-\phi(\bar{z})/2}S_\beta(\bar{z})e^{i\frac{\alpha'}{2}p\cdot D\cdot X(\bar{z})}\lambda \otimes \sigma_3\sigma_1, \\
V_C^{(-\frac{3}{2},-\frac{1}{2})}(z,\bar{z}) &= (P_-\slashed{C}_{(n)}M_p)^{\alpha\beta}e^{-3\phi(z)/2}S_\alpha(z)e^{i\frac{\alpha'}{2}p\cdot X(z)}e^{-\phi(\bar{z})/2}S_\beta(\bar{z})e^{i\frac{\alpha'}{2}p\cdot D\cdot X(\bar{z})}\lambda \otimes \sigma_1 \quad (6)
\end{aligned}
$$

This provides a very interesting selection rule for all non-BPS brane amplitudes involving world volume fermion fields.

We highlight the crucial fact that the correlation function between two spin operators and one world sheet fermion field of non-BPS branes (even with some more fermion fields and/or currents coming from scalar /gauge in different pictures) is non-zero however due to postulating CP factors we realize that the amplitude of two fermion fields and an arbitrary

numbers of gauge/scalar (even in different pictures) and one tachyon makes no sense in the world volume of non-BPS branes.

As an instance let us talk about the amplitude of two closed string Ramond-Ramond and one tachyon. We claim this amplitude has zero value so there is no need to carry out all the correlation functions including four spin operators and one fermion field. Here is its argument.

We believe that the correlation function between four spin operators and one world sheet fermion field of non-BPS branes is non-zero, however, due to postulating CP factors for $C^{-1}C^{-1}T^0$ $(\text{Tr}\,(\sigma_3\sigma_1\sigma_3\sigma_1\sigma_1) = 0)$ we realize that the amplitude of two closed string Ramond-Ramond (even with their different pictures with the same or different chirality) and one tachyon makes no sense in the world volume of non-BPS branes.

The conclusion is that there is no coupling for any ordering of CCT amplitude in the world volume of non-BPS branes, so there should not be any coupling between them in field theory either.

2.1   The four point $< V_C V_T V_{\bar\psi} V_\psi >$ amplitude in IIB

From now on we would like to compute the amplitude of one closed string Ramond-Ramond (which moves in the bulk), two fermion fields and one tachyon in the world volume of non-BPS branes.

Motivation for carrying out this S-matrix is to actually obtain all infinite higher derivative corrections of two fermions and two tachyons to all orders in α' and to fix precisely their coefficients as well. The extremely important point should be made is that, for sure these corrections can not be applied for brane anti brane systems (one might go through [30] for further details), thus these corrections which we are getting to derive are just related to two fermions and two tachyons of non-BPS branes and not to brane-anti brane systems.

Some remarks about our amplitude $CT\bar\psi\psi$ must be highlighted.

The first fact is that both two fermion fields here should have different chiralities which means that our calculation makes sense just for IIB super string theory.

This amplitude has zero result for IIA and the reason for this sharp conclusion is as follows:

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The multiplication of two spin operators with the same chirality gives us a vector (the same thing so happens for two spin operators of closed string RR). The multiplication of these two vectors with one fermion field (the vertex of tachyon in zero picture consists of a vector in space-time) gives us three vectors and the multiplication of three vectors can not give rise a singlet.

Hence we immediately conclude that the correlation function of four spin operators (with the same chirality) and one fermion field has zero value.

Therefore $< V_C V_T V_{\bar{\psi}} V_\psi >$ in IIA has zero result which means that neither there are first order couplings between one RR, two fermions and one tachyon in IIA nor infinite higher derivative corrections .

Rather than the multiplication of two spin operators with different chirality (in IIB) gives us a singlet (identity matrix) and from RR we get a vector. Thus essentially the multiplication of a vector (tachyon in zero picture) and an identity matrix (two spin operators with different chiralities) and a vector (from RR) gives us a singlet. Therefore $< V_C V_T V_{\bar{\psi}} V_\psi >$ just makes sense for IIB string theory.

Hence $< V_C V_T V_{\bar{\psi}} V_\psi >$ amplitude should be looked as follows

$$\mathcal{A}^{CT\bar{\psi}\psi} \sim \int dx_1 dx_2 dx_3 dx_4 dx_5 \langle V_T^{(0)}(x_1) V_{\bar{\psi}}^{(-1/2)}(x_2) V_\psi^{(-1/2)}(x_3) V_C^{(-1)}(x_4, x_5)\rangle, \quad (7)$$

We just need to take into account one ordering of the amplitude so we choose it to be $\mathrm{Tr}\,(\lambda_1\lambda_2\lambda_3)$ as usual. Having substituted our vertices in the S-matrix, some extraordinary tool is needed.

Indeed one has to look for the correlation function of one fermion field and four spin operators in ten dimensions of space time (in order to have non zero result in IIB string theory three spin operator have to have the same chirality and the fourth spin operator must carry different chirality). This correlator at one loop level has been derived in [54] but we are looking for it at tree level computations. Therefore in order to find it out at tree level, one has to set all theta functions to an identity matrix. Another important point is in order.

In order to have the full result of the amplitude of $< V_C V_T V_{\bar{\psi}} V_\psi >$, the special gauge fixing (namely gauge fixing on the location of open strings) is necessary, in such a way

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that the needed correlator would be found as :

$$< \psi^a(x_1) : S_\alpha(x_4) : S_\beta(x_5) : S_\gamma(x_2) : S^{\dot{\delta}}(x_3) > \quad = \quad \frac{(x_{45}x_{42}x_{52})^{-3/4}(I^a_{\alpha\beta\gamma\dot{\delta}})}{\sqrt{2}(x_{12}x_{13}x_{14}x_{15})^{1/2}(x_{43}x_{53}x_{23})^{1/4}}$$

where

$$
\begin{aligned}
I^a_{\alpha\beta\gamma\dot{\delta}} \quad = \quad & \Big[\frac{C_\gamma{}^{\dot{\delta}}}{x_{23}}(\gamma^a\,C)_{\alpha\beta}x_{13}x_{42}x_{52} + \frac{C_\alpha{}^{\dot{\delta}}}{x_{43}}(\gamma^a\,C)_{\beta\gamma}x_{13}x_{45}x_{42} - \frac{C_\beta{}^{\dot{\delta}}}{x_{53}}(\gamma^a\,C)_{\alpha\gamma}x_{13}x_{45}x_{52} \\
& -\frac{1}{2}(\gamma^\nu\gamma^a\,C)_\gamma{}^{\dot{\delta}}(\gamma_\nu\,C)_{\alpha\beta}x_{14}x_{52} + \frac{1}{2}\,(\gamma^\nu\gamma^a\,C)_\alpha{}^{\dot{\delta}}\,(\gamma_\nu\,C)_{\beta\gamma}x_{12}x_{45}\Big]
\end{aligned}
\tag{8}
$$

Having applied above correlator inside the amplitude, we get to

$$
\begin{aligned}
\mathcal{A}^{CT\bar{\psi}\psi} \quad \sim \quad & \int dx_1dx_2dx_3dx_4dx_5\,(P_-\!\!\!\!/\!\!\!H_{(n)}M_p)^{\alpha\beta}\bar{u}_1^\gamma u_2^{\dot{\delta}}(x_{23}x_{24}x_{25}x_{34}x_{35}x_{45})^{-1/4}\mathrm{Tr}\,(\sigma_1\sigma_3 I\sigma_3\sigma_1)I_1 \\
& \times(\alpha' ik_{1a}) < \psi^a(x_1) : S_\alpha(x_4) : S_\beta(x_5) : S_\gamma(x_2) : S^{\dot{\delta}}(x_3) > \mathrm{Tr}\,(\lambda_1\lambda_2\lambda_3),
\end{aligned}
\tag{9}
$$

where $x_{ij} = x_i - x_j, x_4 = z = x + iy, x_5 = \bar{z} = x - iy$, and

$$I_1 \quad = \quad <: e^{\alpha' ik_1.X(x_1)} : e^{\alpha' ik_2.X(x_2)} : e^{\alpha' ik_3.X(x_3)} : e^{i\frac{\alpha'}{2}p.X(x_4)} : e^{i\frac{\alpha'}{2}p.D.X(x_5)} :>$$

Concerning Wick theorem we find

$$I_1 = |x_{12}|^{\alpha'^2 k_1.k_2}|x_{13}|^{\alpha'^2 k_1.k_3}|x_{14}x_{15}|^{\frac{\alpha'^2}{2}k_1.p}|x_{23}|^{\alpha'^2 k_2.k_3}|x_{24}x_{25}|^{\frac{\alpha'^2}{2}k_2.p}|x_{34}x_{35}|^{\frac{\alpha'^2}{2}k_3.p}|x_{45}|^{\frac{\alpha'^2}{4}p.D.p}$$

Setting the correlators in the integrand, we realize the fact that the amplitude has the property of SL(2,R) invariance. Let us define the Mandelstam variables as

$$s \quad = \quad \frac{-\alpha'}{2}(k_1 + k_3)^2, \quad t = \frac{-\alpha'}{2}(k_1 + k_2)^2, \quad u = \frac{-\alpha'}{2}(k_3 + k_2)^2$$

Performing the gauge fixing on the location of open strings as $(x_1 = 0, x_2 = 1, x_3 = \infty)$, we may find the final form of the amplitude to be ready to take integrations on the location of closed string as follows

$$\mathcal{A}^{CT\bar{\psi}\psi} = (P_-\slashed{H}_{(n)}M_p)^{\alpha\beta}(2\alpha'ik_{1a})\bar{u}_1^\gamma u_2^{\dot\delta}\frac{1}{\sqrt{2}}\int dzd\bar{z}|z|^{2t+2s-1}|1-z|^{2t+2u-3/2}(z-\bar{z})^{-2(t+s+u)-3/2}$$

$$\times\Big[C_\gamma^{\dot\delta}(\gamma_a C)_{\alpha\beta}|1-z|^2 - C_\alpha^{\dot\delta}(z-\bar{z})(1-z)(\gamma^a C)_{\beta\gamma} + C_\beta^{\dot\delta}(\gamma^a C)_{\alpha\gamma}(z-\bar{z})(1-\bar{z})$$

$$-\frac{1}{2}(\gamma^\nu\gamma^a C)_\gamma^{\dot\delta}(\gamma_\nu C)_{\alpha\beta}z(1-\bar{z}) - \frac{1}{2}(\gamma^\nu\gamma^a C)_\alpha^{\dot\delta}(\gamma_\nu C)_{\beta\gamma}(z-\bar{z})\Big]\mathrm{Tr}\,(\lambda_1\lambda_2\lambda_3),(10)$$

One important check of our amplitude and in particular our correlators is indeed producing all the leading singularities in which our computation takes care of it. In fact by sending x_1 to x_2 we get to the correct correlator of four spin operators where two of them carry different chirality [54] and this is one more test in favor of our computations.

The double integrals should be performed on the closed string position to actually get the entire result (further details should be found in [55], [23]). Hence the final result for our S-matrix is gotten as:

$$\mathcal{A}^{CT\bar{\psi}\psi} = \mathrm{Tr}\,(\lambda_1\lambda_2\lambda_3)(P_-\slashed{H}_{(n)}M_p)^{\alpha\beta}(2\alpha'ik_{1a})\bar{u}_1^\gamma u_2^{\dot\delta}\frac{1}{\sqrt{(2)}}\Big[C_\gamma^{\dot\delta}(\gamma_a C)_{\alpha\beta}L_1 + \Big(-C_\alpha^{\dot\delta}(\gamma^a C)_{\beta\gamma}$$

$$+C_\beta^{\dot\delta}(\gamma^a C)_{\alpha\gamma}\Big)\frac{-sL_2}{(-s-u+\frac{1}{4})} + \frac{1}{2}\Big(C_\alpha^{\dot\delta}(\gamma^a C)_{\beta\gamma} + C_\beta^{\dot\delta}(\gamma^a C)_{\alpha\gamma}\Big)L_3 + \frac{1}{2}(\gamma^\nu\gamma^a C)_\gamma^{\dot\delta}(\gamma_\nu C)_{\alpha\beta}$$

$$\times\frac{(-u-\frac{1}{4})}{(-t-u-\frac{1}{4})}L_1 + \Big(-\frac{1}{2}(\gamma^\nu\gamma^a C)_\alpha^{\dot\delta}(\gamma_\nu C)_{\beta\gamma} - \frac{1}{4}(\gamma^\nu\gamma^a C)_\gamma^{\dot\delta}(\gamma_\nu C)_{\alpha\beta}\Big)L_2\Big] \qquad (11)$$

with

$$L_1 = (2)^{-2(t+s+u)-3/2}\pi\frac{\Gamma(-u-\frac{1}{4})\Gamma(-s+\frac{1}{2})\Gamma(-t)\Gamma(-t-s-u-\frac{1}{4})}{\Gamma(-u-t-\frac{1}{4})\Gamma(-t-s+\frac{1}{2})\Gamma(-s-u+\frac{1}{4})},$$

$$L_2 = (2)^{-2(t+s+u)-1/2}\pi\frac{\Gamma(-u+\frac{1}{4})\Gamma(-s)\Gamma(-t+\frac{1}{2})\Gamma(-t-s-u+\frac{1}{4})}{\Gamma(-u-t+\frac{3}{4})\Gamma(-t-s+\frac{1}{2})\Gamma(-s-u+\frac{1}{4})},$$

$$L_3 = (2)^{-2(t+s+u)+1/2}\pi\frac{\Gamma(-u+\frac{3}{4})\Gamma(-s+\frac{1}{2})\Gamma(-t)\Gamma(-t-s-u+\frac{3}{4})}{\Gamma(-u-t+\frac{3}{4})\Gamma(-t-s+\frac{1}{2})\Gamma(-s-u+\frac{5}{4})},$$

$$(12)$$

By postulating the internal degrees of freedom (CP factor) for all strings in the presence of non-BPS branes, we expect to get the same answer for our amplitude in the other

different pictures. Hence we expect to get exactly the same result for the amplitude of $< V_C^{-1}(z,\bar z) V_T^{-1}(x_1) V_{\bar\Psi}^{1/2}(x_2) V_\Psi^{-1/2}(x_3) >$.

Note also that by removing tachyons we are set to BPS branes in which they do not carry CP factor thus the kinetic term of fermion fields (even in the presence of non-BPS branes) is just

$$2\pi\alpha'\mathrm{Tr}\,(\bar\Psi\gamma^\mu D_\mu\Psi)$$

and in particular it does not carry internal degrees of freedom in field theory side. Therefore the kinetic term of fermion fields keeps fixed even in the world volume of non-BPS branes in the DBI effective action.

By taking a look at the final result of our amplitude, we reveal that it does have non-zero couplings for diverse cases. One good remark should be made as follows.

Here we are not dealing with massless strings so the expansion as it stands, is not low energy expansion. How can we expand the amplitude?

The answer is that, the amplitude should be expanded such that all the singularities (massless/tachyonic poles) are indeed produced in comparison with the effective field theory arguments.

Let us mention some hints from string theory 's point of view. In the last section we proved that by making use of the internal degrees of freedom, neither there are couplings between two fermion fields and one tachyon nor between two tachyons and one fermion field.

Having set this remark, we observe that there is no any kind of tachyon/fermion pole for this amplitude. Therefore we should not expect to have any s, t-channel pole in this amplitude.

Hence, by applying this remark to all the Gamma functions of our amplitude we might discover that both t, s must not send to $0, \frac{1}{2}$. We will talk about the other conditions for our amplitude later on.

The other fact which must be highlighted is that by applying all CP factors we have checked that there is no any double poles including fermion-fermion, fermion-tachyon, tachyon-scalar, fermion-gauge (scalar), tachyon-tachyon and gauge-gauge (scalar).

For example one may talk about the coupling between two fermions and one scalar from left hand side and the coupling between two tachyons and one scalar from the middle and the coupling between one RR and one tachyon from right hand side. However this diagram does not have any contribution given the fact that coupling between two tachyons and one scalar is zero (although the amplitude of two tachyons and one scalar does have non zero CP factor as shown by $\mathrm{Tr}\,(\sigma_3\sigma_2\sigma_1) = -2i$).

Figure 1 : The Feynman diagrams corresponding to the four point function of $CT_1\bar{\Psi}_2\Psi_3$.

On the other hand (as it has been drawn) this amplitude should have infinite tachyon poles for $n = p + 1$ case, as there are non-zero couplings between two fermions and two tachyons in the world volume of non-BPS branes (the CP factor for $A^{\bar{\psi}^{-1/2}\psi^{-1/2}T^{-1}T^0}$ has also non zero value $\mathrm{Tr}\,(\sigma_3 I \sigma_2 \sigma_1) = -2i$). Also the amplitude does have infinite contact interactions which we are not going to explore them in this paper.

Here we insist of performing direct computations to see whether or not the universal conjecture for infinite higher derivative corrections of type II string theory works out (even for non-BPS fermionic amplitudes).

In order to derive all non-Abelian and the infinite higher derivative corrections of two tachyons and two fermions (to all orders in α') and to produce all the infinite $u + s' + t'$ tachyon poles in string amplitude, one should have the expansion of the amplitude of two tachyons and two fermions and should also know the final result of the amplitude of $(A^{\bar{\psi}\psi TT})$ to be able to precisely produce all the coefficients of $a_{n,m}, b_{n,m}$ in field theory side.

Note that essentially we need to compare these coefficients at each order of α' with string coefficients in favor of obtaining all the infinite higher derivative corrections of two tachyons and two fermions in the world volume of non-BPS branes.

Namely one has to have the complete form of both amplitudes. Thus all the following orderings must be taken into account :

$$\text{Tr}\,(\lambda_1\lambda_2\lambda_3\lambda_4),\,\text{Tr}\,(\lambda_1\lambda_2\lambda_4\lambda_3),\,\text{Tr}\,(\lambda_1\lambda_3\lambda_2\lambda_4)$$
$$\text{Tr}\,(\lambda_1\lambda_3\lambda_4\lambda_2),\,\text{Tr}\,(\lambda_1\lambda_4\lambda_3\lambda_2),\,\text{Tr}\,(\lambda_1\lambda_4\lambda_2\lambda_3),$$

The other fact which must be regarded is that the final result of the open string amplitudes (like $A^{\bar\psi^{-1/2}\psi^{-1/2}T^{-1}T^0}$) must respect all symmetries. Here the final answer should respect all symmetries related to two fermions and two tachyons amplitude, basically it has to be totally anti symmetric with respect to interchanging fermions, furthermore it should be symmetric under interchanging two tachyons .

In order to respect these symmetries, one has to take into account all Chan-Paton factors and in particular consider all six possible orderings of this amplitude (for four point open super strings).

Now let us first carry out this amplitude for $\text{Tr}\,(\lambda_1\lambda_2\lambda_3\lambda_4)$ ordering as:

$$
\begin{aligned}
\mathcal{A}^{\bar\Psi,\Psi,T,T} &= (-8T_p 2^{1/2})\int dx_1 dx_2 dx_3 dx_4 \text{Tr}\,\langle V_{\bar\Psi}^{(-1/2)}(x_1)V_{\Psi}^{(-1/2)}(x_2)V_T^{(-1)}(x_3)V_T^{0}(x_4)\rangle \\
&= (-8T_p)\int dx_1 dx_2 dx_3 dx_4 |x_{12}|^{\alpha'^2 k_1\cdot k_2}|x_{13}|^{\alpha'^2 k_1\cdot k_3}|x_{14}|^{\alpha'^2 k_1\cdot k_4}|x_{23}|^{\alpha'^2 k_2\cdot k_3}|x_{24}|^{\alpha'^2 k_2\cdot k_4} \\
&\quad \times |x_{34}|^{\alpha'^2 k_3\cdot k_4} x_{12}^{-1} x_{13}^{-\frac{1}{2}}(x_{41}x_{42})^{-\frac{1}{2}}x_{23}^{-\frac{1}{2}}\text{Tr}\,(\lambda_1\lambda_2\lambda_3\lambda_4)\text{Tr}\,(\sigma_3 I\sigma_2\sigma_1)(\alpha' ik_{4a})\bar u_1^A(\gamma^a)_{AB}u_2^B
\end{aligned}
$$

One can show that the amplitude is now $SL(2,R)$ invariant. To get the final answer in terms of Gamma function one has to carry out gauge fixing as $(x_1,x_2,x_3,x_4)=(0,x_2,1,\infty)$ where at the end we get to

$$
\begin{aligned}
\mathcal{A}^{\bar\Psi,\Psi,T,T} &= (-8T_p)\text{Tr}\,(\sigma_3 I\sigma_2\sigma_1)\bar u_1^A(\gamma^a)_{AB}u_2^B(\alpha' ik_{4a})\int_0^1 dx_2 x_2^{-2t-1}(1-x_2)^{-2u-1}\text{Tr}\,(\lambda_1\lambda_2\lambda_3\lambda_4) \\
&= (-8T_p)\bar u_1^A(\gamma^a)_{AB}u_2^B(\alpha' ik_{4a})\text{Tr}\,(\lambda_1\lambda_2\lambda_3\lambda_4)\text{Tr}\,(\sigma_3 I\sigma_2\sigma_1)\frac{\Gamma(-2t)\Gamma(-2u)}{\Gamma(-2t-2u)} \qquad (13)
\end{aligned}
$$

However one has to find out all the other 5 possible orderings as well. For instance for $\text{Tr}\,(\lambda_1\lambda_3\lambda_2\lambda_4)$ the corrected gauge fixing is $(x_1,x_3,x_2,x_4)=(0,x_3,1,\infty)$ also note that in all possible orderings the picture of open strings should be kept fixed.

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By extracting the CP factors the final and complete form of the amplitude is

$$
\begin{aligned}
\mathcal{A}^{\bar{\Psi},\Psi,T,T} &= (-8T_p)\bar{u}_1^A(\gamma_{AB}^a)u_2^B(\alpha'ik_{4a})\text{Tr}\,(\sigma_3 I\sigma_2\sigma_1)\Big(l_1\frac{\Gamma(-2t)\Gamma(-2u)}{\Gamma(-2t-2u)} - l_2\frac{\Gamma(-2t)\Gamma(-2s)}{\Gamma(-2t-2s)} \\
&\quad +il_3\frac{\Gamma(-2u)\Gamma(-2s)}{\Gamma(-2u-2s)}\Big)
\end{aligned} \tag{14}
$$

where l_1, l_2, l_3 are defined as

$$
\begin{aligned}
l_1 &= \frac{1}{2}\Big(\text{Tr}\,(\lambda_1\lambda_2\lambda_3\lambda_4) + \text{Tr}\,(\lambda_1\lambda_4\lambda_3\lambda_2)\Big) \\
l_2 &= \frac{1}{2}\Big(\text{Tr}\,(\lambda_1\lambda_2\lambda_4\lambda_3) + \text{Tr}\,(\lambda_1\lambda_3\lambda_4\lambda_2)\Big) \\
l_3 &= \frac{1}{2}\Big(\text{Tr}\,(\lambda_1\lambda_3\lambda_2\lambda_4) + \text{Tr}\,(\lambda_1\lambda_4\lambda_2\lambda_3)\Big)
\end{aligned}
$$

The very non trivial question would be related to the expansion of the amplitude, basically how to expand the amplitude such that all massless poles are not removed. Making use of the on-shell condition

$$
s + t + u = -\frac{1}{2}
$$

we might realize that all Mandelstam variables should not be sent to zero. Remember there is a non zero coupling between two fermions and one gauge field, moreover there is a non zero coupling between two tachyons and one gauge field. Thus we realize that there must be a massless t-channel pole for this amplitude.

Also note that the amplitude must be symmetric with respect to interchanging s- and u (under interchanging both tachyons) so the correct expansion which satisfies on-shell condition is indeed

$$
t \to 0, \qquad s, u \to -\frac{1}{4} \tag{15}
$$

Let us write down the above momentum expansion just in terms of momenta,(see [23, 44])

$$
(k_1 + k_2)^2, k_1 \cdot k_3, k_2 \cdot k_3 \to 0
$$

The above relation is the momentum expansion that we have looked for. Using $u' = u + 1/4 = -\alpha'k_2 \cdot k_3$ and $s' = s + 1/4 = -\alpha'k_1 \cdot k_3$ we can derive new on-shell relation as $s' + t + u' = 0$.

One can make use of the new relation $(s' + t + u' = 0)$ and also use the above change of variables to actually derive the final form of the amplitude as follows:

$$
\mathcal{A}^{\bar{\Psi}_1,\Psi_2,T_3,T_4} = (-8T_p)\bar{u}_1^A(\gamma_{AB}^a)u_2^B(\alpha' i k_{4a})\text{Tr}\,(\sigma_3 I \sigma_2 \sigma_1)\Big(l_1 \frac{\Gamma(2u' + 2s')\Gamma(\frac{1}{2} - 2u')}{\Gamma(\frac{1}{2} + 2s')}
$$
$$
- l_2 \frac{\Gamma(2u' + 2s')\Gamma(\frac{1}{2} - 2s')}{\Gamma(\frac{1}{2} + 2u')} + i l_3 \frac{\Gamma(\frac{1}{2} - 2s')\Gamma(\frac{1}{2} - 2u')}{\Gamma(1 - 2s' - 2u')}\Big) \tag{16}
$$

If we expand our amplitude around (15) we eventually get

$$
\mathcal{A}^{\bar{\Psi}_1,\Psi_2,T_3,T_4} = -8T_p\bar{u}_1^A(\gamma_{AB}^a)u_2^B(\alpha' i k_{4a})\text{Tr}\,(\sigma_3 I \sigma_2 \sigma_1) \tag{17}
$$
$$
\Big(\frac{l_1 - l_2}{-2t} + \sum_{n,m=0}^{\infty}\Big[a_{n,m}(l_1 u'^n s'^m - l_2 s'^n u'^m) + i l_3 b_{n,m}(s'^n u'^m + s'^m u'^n)\Big]\Big)
$$

Ultimately after using the expansion in (15) we are able to derive all the correct coefficients of $a_{n,m}, b_{n,m}$ as follows

$$
a_{0,0} = 2\ln(2),\, b_{0,0} = \frac{\pi}{2},\, a_{1,1} = -12\zeta(3) + \frac{32}{3}\ln(2)^3 + \frac{4\pi^2}{3}\ln(2),\, b_{1,1} = \frac{\pi}{3}(-\pi^2 + 24\ln(2)^2),
$$
$$
a_{1,0} = \frac{2\pi^2}{3} + 4\ln(2)^2,\, b_{1,0} = 2\pi\ln(2),\, a_{0,1} = -\frac{\pi^2}{3} + 4\ln(2)^2,
$$
$$
a_{2,0} = 8\zeta(3) + \frac{16}{3}\ln(2)^3 + \frac{8\pi^2}{3}\ln(2),\, b_{2,0} = \frac{\pi}{3}(\pi^2 + 12\ln(2)^2)
$$
$$
a_{0,2} = 8\zeta(3) + \frac{16}{3}\ln(2)^3 - \frac{4\pi^2}{3}\ln(2),\, b_{1,2} = \frac{4\pi}{3}(12\ln(2)^3 - 3\zeta(3)),
$$
$$
b_{3,0} = \frac{4\pi}{3}(\pi^2\ln(2) + 4\ln(2)^3 + 6\zeta(3)),\, \cdots \tag{18}
$$

Note that $b_{n,m}$'s coefficients are symmetric. The massless gauge t-channel pole can just be resulted by taking into account all the kinetic terms of fermions, gauges and tachyons.

$$
- T_p\frac{2\pi\alpha'}{2}\text{Tr}\,\Big(\bar{\Psi}\gamma^a D_a\Psi + D_a T D^a T - \frac{(2\pi\alpha')}{2}F_{ab}F^{ba}\Big) \tag{19}
$$

where

$$
D_a\Psi = \partial_a\Psi - i[A^a, \Psi],\, D_a T = \partial_a T - i[A^a, T],\, D_i\Psi = \partial_i\Psi - i[\phi^i, \Psi],
$$

In the other words the massless t-channel gauge pole should be produced by considering the following rule

$$
V^{a,\alpha}(\bar{\Psi}_1, \Psi_2, A)G^{ab,\alpha\beta}(A)V^{b,\beta}(A, T_3, T_4)
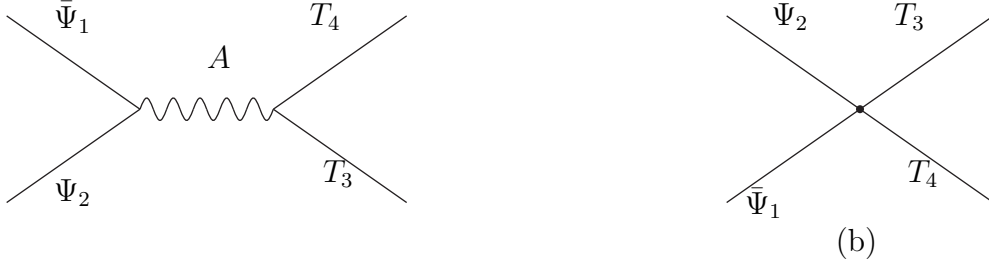$$

(b)

Figure 2 : The Feynman diagrams corresponding to the amplitude of $\bar{\Psi}_1 \Psi_2 T_3 T_4$.

such that

$$
\begin{aligned}
G^{ab,\alpha\beta}(A) &= \frac{i\delta^{ab}\delta^{\alpha\beta}}{(2\pi\alpha')^2 T_p t} \\
V^{a,\alpha}(\bar{\Psi}_1, \Psi_2, A) &= T_p(2\pi\alpha')\bar{u}_1^A \gamma_{AB}^a u_2^B \left(\text{Tr}\,(\lambda_1\lambda_2\lambda^\alpha) - \text{Tr}\,(\lambda_2\lambda_1\lambda^\alpha) \right) \\
V^{b,\beta}(A, T_3, T_4) &= T_p(2\pi i\alpha')(k_3^b - k_4^b)\left(\text{Tr}\,(\lambda_4\lambda_3\lambda^\beta) - \text{Tr}\,(\lambda_3\lambda_4\lambda^\beta) \right)
\end{aligned}
\tag{20}
$$

The other question which should be addressed is how to produce the contact terms. Let us write down the zeroth order of contact terms of the amplitude as

$$
\mathcal{A}^{\bar{\Psi}_1, \Psi_2, T_3, T_4} = -8T_p(2i)\bar{u}_1^A(\gamma_{AB}^a)u_2^B(\alpha' i k_{4a})\left[a_{0,0}(l_1 - l_2) + 2il_3 b_{0,0}\right]
$$

The above contact interactions can be derived by writing down some suitable couplings.

It is worth emphasizing that these new couplings must have the same order like the kinetic interaction of the fermion fields. The momentum conservation along the world volume of brane $(k_1 + k_2 + k_3 + k_4)^a = 0$ as well as on shell conditions for fermion fields $\gamma^a k_{1a}\bar{u}^A = \gamma^a k_{2a}u^B = 0$ should have been applied. Hence contact interaction at first order can be summarized as

$$
\alpha' T_p \text{Tr}\left(a_{0,0}\left(\bar{\Psi}\gamma^a\Psi T D_a T - \bar{\Psi}\gamma^a\Psi D_a T T \right) + ib_{0,0}\left(\bar{\Psi}\gamma^a T\Psi D_a T - \bar{\Psi}\gamma^a D_a T\Psi T \right) \right)
\tag{21}
$$

It is not a big deal to show that for Abelian groups and making use of just $(a_{0,1} - a_{1,0})$ term, the first non zero term in (17) can be precisely produced by expanding the effective action that appeared in page two and taking the following coupling (for the expansion see [44]).

$$
2\pi^2\alpha'^2 T_p \bar{\Psi}\gamma^b\partial^a\Psi\partial_b T\partial_a T
$$

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Now we come to one of our main goals which is producing non-Abelian interactions of the scattering amplitude of one RR, one tachyon and two world volume fermion fields.

The coefficient of $\bar{u}_1^A(\gamma_{AB}^a)u_2^B(i\alpha'k_{1a})$ is the over all coefficient for all terms inside the final result of the amplitude of $CT\bar{\psi}\psi$. According to the prescription which has been given in [49, 23], we expect to actually have explored all the non-Abelian couplings of two fermions and two tachyons in the world volume of non-BPS branes as :

$$
\begin{aligned}
\mathcal{L}^{n,m} &= \frac{\alpha'}{2}\alpha'^{n+m}T_p\left(a_{n,m}\mathrm{Tr}\left[\mathcal{D}_{nm}\left(\bar{\Psi}\gamma^a\Psi TD_aT\right) + \mathcal{D}_{nm}\left(TD_aT\bar{\Psi}\gamma^a\Psi\right)\right.\right. \\
&\quad \left. -\mathcal{D}_{nm}\left(\bar{\Psi}\gamma^a\Psi D_aTT\right) - \mathcal{D}_{nm}\left(D_aTT\bar{\Psi}\gamma^a\Psi\right) + h.c\right] \\
&\quad +ib_{n,m}\mathrm{Tr}\left[\mathcal{D}'_{nm}\left(\bar{\Psi}\gamma^aT\Psi D_aT\right) + \mathcal{D}'_{nm}\left(\Psi D_aT\bar{\Psi}\gamma^aT\right)\right. \\
&\quad \left.\left. -\mathcal{D}'_{nm}\left(\bar{\Psi}\gamma^aD_aT\Psi T\right) - \mathcal{D}'_{nm}\left(\Psi T\bar{\Psi}\gamma^aD_aT\right) + h.c\right]\right)
\end{aligned}
$$
(22)

in such a way that the higher derivative operators $\mathcal{D}_{nm}$ and $\mathcal{D}'_{nm}$ are used to be defined

$$
\begin{aligned}
\mathcal{D}_{nm}(EFGH) &\equiv D_{b_1}\cdots D_{b_m}D_{a_1}\cdots D_{a_n}EFD^{a_1}\cdots D^{a_n}GD^{b_1}\cdots D^{b_m}H \\
\mathcal{D}'_{nm}(EFGH) &\equiv D_{b_1}\cdots D_{b_m}D_{a_1}\cdots D_{a_n}ED^{a_1}\cdots D^{a_n}FGD^{b_1}\cdots D^{b_m}H
\end{aligned}
$$

In fact the above couplings are non-Abelian extensions at the order of $(\alpha')^{1+n+m}$ of $a_{0,0}$ and $b_{0,0}$. One highly important remark about our notation is in order.

In the term $\mathcal{D}'_{nm}\left(\Psi D_aT\bar{\Psi}\gamma^aT\right)$ in (22), first we need to apply all the higher derivative terms on it and then one should rewrite it as $\left(\bar{\Psi}\gamma^aT\Psi D_aT\right)$.

This is another check in favor of universality conjecture for higher derivative corrections to all orders in α' for all non-BPS and BPS branes [49].

2.2 An infinite number of tachyon poles of $CT\bar{\Psi}\Psi$ for $p+1=n$ case

In order to show that we have derived all the infinite non-Abelian couplings of two fermions and two tachyons in the world volume of non-BPS branes (22), in this section we are going to use those couplings to actually produce all the infinite tachyon poles of string amplitude.

Extracting the traces, we just write down all the singularities of the string amplitude as

$$
\begin{aligned}
\mathcal{A}^{CT\bar{\Psi}\Psi} &= \frac{32i\pi^2\mu'_p\beta'}{4(p+1)!}\bar{u}_1^A(\gamma^a_{AB})u_2^B\mathrm{Tr}\,(\lambda_1\lambda_2\lambda_3)\epsilon^{a_0\cdots a_p}H_{a_0\cdots a_p} \\
&\times(-2i\alpha'k_{1a})\sum_{n,m=0}^{\infty}\frac{e_{n,m}[s'^nt'^m - t'^ns'^m]}{(-t'-s'-u)}
\end{aligned}
\tag{23}
$$

where we have normalized the string amplitude by $\frac{\beta'\mu'_p}{(2\pi)^{1/2}}$ and μ'_p is the RR charge of branes and β' is the normalization of Wess-Zumino actions for non-BPS branes (further details can be understood in [23]).

Note that we have expanded the amplitude of $CT\bar{\Psi}\Psi$ out such that both conditions

$$
t + s + u = -p^a p_a - \frac{1}{4}
$$

which is momentum conservation along the world volume of brane and the condition comes from Gamma function $(-t - s - u - \frac{1}{4} = 0)$ could simultaneously be satisfied.

It is argued in [23, 30] that for non-BPS branes and brane -anti brane systems the quantity $p^a p_a$ must tend to $\frac{1}{4}, 0$ accordingly. Also note that from our S-matrix it is kind of obvious that the expansion of fermionic amplitudes in the presence of tachyons is certainly different from the expansion of the amplitudes including RR, gauge/scalar in the presence of tachyons.

The reason can be understood by taking a look at the concluded operator product expansions from super ghost charges in such a way that after considering all symmetries here we do have $\Gamma(-t-s-u-\frac{1}{4})$ in the amplitude rather than having usual $\Gamma(-t-s-u-\frac{1}{2})$ for bosonic amplitudes.

In order to derive all the infinite tachyon poles of the string amplitude one must use the following Feynman rule :

$$
\mathcal{A} = V^\alpha(C_p, T)G^{\alpha\beta}(T)V^\beta(T, T_1, \bar{\Psi}_2, \Psi_3)
\tag{24}
$$

such that

$$
\begin{aligned}
G^{\alpha\beta}(T) &= \frac{i\delta^{\alpha\beta}}{(2\pi\alpha')T_p(-k^2 - m^2)} \\
V^\alpha(C_p, T) &= 2i\mu'_p\beta'(2\pi\alpha')\frac{1}{(p+1)!}\epsilon^{a_0\cdots a_p}H_{a_0\cdots a_p}\mathrm{Tr}\,(\Lambda^\alpha)
\end{aligned}
\tag{25}
$$

Notice that the propagator can be written as $\frac{i\delta^{\alpha\beta}}{(2\pi\alpha')T_p(-s'-t'-u)}$ and $\mathrm{Tr}\,(\Lambda^\alpha)$ is just non-zero for Abelian Λ^α.

Now the $V^\beta(T, T_1, \bar{\Psi}_2, \Psi_3)$ should be looked for by making use of the derived higher derivative non-abelian couplings of (22).

To obtain two on-shell fermions, one on-shell tachyon and one off-shell tachyon's vertex operator, we need to work out with two different orderings of open strings as follows:

$$
\mathrm{Tr}\,(\lambda_2\lambda_3\lambda_1\lambda_\beta), \qquad \mathrm{Tr}\,(\lambda_2\lambda_3\lambda_\beta\lambda_1)
\tag{26}
$$

where β is related to the Abelian tachyon. Let us focus firstly on $a_{n,m}$ such that by taking the following term $\mathrm{Tr}\,\left(a_{n,m}\bar{\Psi}\gamma^a\Psi T D_a T\right)$, we are able to derive the following vertices

$$
\begin{aligned}
&a_{n,m}(k.k_2)^m(k_1.k_2)^n\bar{u}\gamma^a u(-ik_{4a}) \\
&+a_{n,m}(k.k_2)^n(k_1.k_2)^m\bar{u}\gamma^a u(-ik_{1a})
\end{aligned}
\tag{27}
$$

where k becomes off-shell tachyon's momentum. Now we need to take into account the hermitian conjugate of this term as well as carrying out the same procedure to all the other terms in (22). Adding all the terms up, we can derive the following vertex (just for the terms including the coefficient of $a_{n,m}$)

$$
\begin{aligned}
&a_{n,m}\bar{u}\gamma^a u\left(ik_{4a} - ik_{1a}\right)\Big\{\Big[-(k.k_2)^m(k_2.k_1)^n + (k_1.k_3)^n(k.k_3)^m - (k_3.k_1)^m(k_2.k_1)^n \\
&+(k_3.k_1)^n(k_1.k_2)^m\Big] - \Big[-(k_2.k_1)^m(k_2.k)^n + (k.k_3)^n(k_1.k_3)^m - (k.k_2)^n(k_3.k)^m \\
&+(k_3.k)^n(k_2.k)^m\Big]\Big\}
\end{aligned}
$$

By using the momentum conservation along the world volume direction, applying equations of motion for fermion fields $\gamma^a k_{2a}\bar{u}^A = \gamma^a k_{3a}u^B = 0$ and also by working out with the

terms having $b_{n,m}$ coefficients (as well as their conjugates) we get to the final answer for $V_\beta^b(\bar{\Psi}_1, \Psi_2, T_3, T)$ as

$$
\begin{aligned}
V_\beta^b(\bar{\Psi}_1, \Psi_2, T_3, T) &= iT_p(\alpha')^{n+m+1}(a_{n,m} + ib_{n,m})\bar{u}_1^A(\gamma^a)_{AB}u_2^B(-\alpha' ik_{1a})\Bigg\{\Bigg[-(k.k_2)^m(k_2.k_1)^n \\
&\quad +(k_1.k_3)^n(k.k_3)^m - (k_3.k_1)^m(k_2.k_1)^n + (k_3.k_1)^n(k_1.k_2)^m + (k_2.k_1)^m(k_2.k)^n \\
&\quad -(k.k_3)^n(k_1.k_3)^m + (k.k_2)^n(k_3.k)^m - (k_3.k)^n(k_2.k)^m\Bigg]\Bigg\}\mathrm{Tr}\,(\lambda_1\lambda_2\lambda_3\lambda_\beta) \quad (28)
\end{aligned}
$$

If we replace (28) into (24), one can explore all the infinite tachyon poles of amplitude in field theory as follows :

$$
32i\beta'\mu_p'\frac{\epsilon^{a_0\cdots a_p}(-ik_{1a})\bar{u}_1^A(\gamma^a)_{AB}u_2^B H_{a_0\cdots a_p}}{(p+1)!(s'+t'+u)}\mathrm{Tr}\,(\lambda_1\lambda_2\lambda_3)\sum_{n,m=0}^{\infty}(a_{n,m}+ib_{n,m})[s'^m t'^n - s'^n t'^m] \quad (29)
$$

Now we would like to explicitly show that the higher derivative terms in (22) are exact. In order to do so, we first remove all the common factors from both string and field theory sides and do check both sides at each order of α'.

At α' order we get the following coefficient in field theory side :

$$
(a_{1,0} - a_{0,1})(s' - t') + (b_{1,0} - b_{0,1})(s' - t') = \pi^2(s' - t')
$$

where we have used the facts that $b_{n,m}$'s are symmetric and in particular the coefficients in (18) have been used. This coefficient is precisely equal to the coefficient in string amplitude as

$$
\pi^2(e_{1,0} - e_{0,1})(s' - t')
$$

At α'^2 order , we get

$$
\begin{aligned}
&(a_{1,1} + ib_{1,1})(s't' - s't') + i(b_{2,0} - b_{0,2})(s'^2 - t'^2) + (a_{2,0} - a_{0,2})[s'^2 - t'^2] \\
&= 4\pi^2 ln(2)(s'^2 - t'^2)
\end{aligned}
$$

which is equivalent to $\pi^2(e_{2,0} - e_{0,2})(s'^2 - t'^2)$ in string amplitude.

Therefore all order α' checks can be made to investigate that not only do we have exactly produced all the higher derivative corrections of two tachyons and two fermions but also all the infinite tachyon poles of $< V_C V_T V_{\bar{\Psi}} V_{\Psi} >$ are precisely resulted.

Thus we come to the fact that these higher derivative couplings of two tachyons and two fermions in (22) are derived in a precise manner. It is worth to mention that these non-BPS corrections can not be used for brane anti brane systems as it is shown in [30].

The final remarks are crucially in order.

Indeed by using $a_{0,0} = 2\ln(2)$ and $b_{0,0} = \pi/2$, we understand the important fact that the non-Abelian couplings at the order of α' could not be matched by applying symmetric trace of non-Abelian couplings of (22).

As it has been argued the above higher derivative corrections can not be used for brane anti brane systems. It would be interesting to actually find out these corrections for brane anti brane systems and also to see whether or not non-Abelian symmetric trace does work at order of α' for brane anti brane systems.

The other open question would be related to solving the ambiguity between ordinary trace and symmetrized trace of the amplitude of one RR and four tachyons in the world volume of brane anti brane systems which is addressed in [23].

Finally we refer to super symmetric action for having included more details and just point out that the complete form of symmetrized action is still unknown [56, 57, 58]. We hope to address these issues in near future.

3   Conclusions

In this paper we have considered the computation of a disk level four point correlation function, including one closed string Ramond-Ramond field in the bulk, one tachyon and two fermion fields ($< V_C V_T V_{\bar{\Psi}} V_{\Psi} >$) in the world volume of non-BPS branes. The aim of this paper was to find out the infinite higher derivative corrections of two fermions and two tachyons to all orders of α' in type IIB super string theory which are derived in (22).

Since there is no correction to the coupling of one Ramond-Ramond and one tachyon as follows

$$2i\beta'\mu_p'(2\pi\alpha') \int_{\Sigma_{p+1}} C_p \wedge DT$$

we have understood that all the infinite tachyon poles must be produced by all the derived corrections in (22), namely we have checked these corrections by producing all the infinite $t' + s' + u$-channel tachyon poles of the string amplitude for $p = n + 1$ case. Basically the presence of possible couplings between these fields in the Wess-Zumino action for non-BPS branes has been investigated. We have also seen that due to internal degrees of freedom of open strings neither there are fermion nor tachyon poles in the amplitude of $CT\bar{\psi}\psi$.

Due to Chan-Paton factors we also argued that there is no coupling between two closed string Ramond-Ramond field and one tachyon in the world volume of non-BPS branes. Therefore this amplitude does not make sense in type II super string theory.

Acknowledgments

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